

Thermodynamic cycle in a cavity optomechanical system

Hou Ian

Institute of Applied Physics and Materials Engineering, University of Macau,
Macau

Abstract. A cavity optomechanical system is initiated by a radiation pressure of a cavity field onto a mirror element acting as a quantum resonator. This radiation pressure can control the thermodynamic character of the mirror to some extent, such as cooling its effective temperature. Here we show that by properly engineering the spectral density of a thermal heat bath that interacts with a quantum system, the evolution of the quantum system can be effectively turned on and off. Inside a cavity optomechanical system, when the heat bath is realized by a multi-mode oscillator modeling of the mirror, this on-off effect translates to infusion or extraction of heat energy in and out of the cavity field, facilitating a four-stroke thermodynamic cycle.

Keywords: cavity optomechanical systems, quantum control, quantum thermodynamics

1. Introduction

The study of cavity electrodynamics roughly began with the discovery of the Fabre-Perot inteferometer, in which two transfective side mirrors sandwich an optical cavity of fixed length, thereby trapping an optical cavity field of designate wavelength inside. When one of the side mirrors is allowed to oscillate, usually by depositing a transfective surface on a micro cantilever, the trapped cavity field will interact with the movable mirror through radiation pressure [1] and other effects induced by the variable cavity length [2].

These kinds of controllable interactions provide some degrees of manipulation to the movable mirror, thus opening the field of cavity optomechanics. In particular, extensive studies have been conducted during the last decade on how to cool down the effective temperature of the mirror [3, 4, 5, 6, 7]. Recently, studies on cavity optomechanical systems have found a wide range of applications such as quadrature squeezing of polariton [8], generation of Kerr nonlinearity [9], and distant state entanglement [10, 11]. However, the quantum thermodynamic aspect of a cavity optomechanical system is less touched upon. In this article, we study the thermodynamic evolution of the cavity field under the influence of radiation pressure feedback from the mirror.

Generally speaking, when a heat bath modeled on an abstract manifold is coupled to a quantum multi-level system, thermodynamic adiabatic processes can be identified, during which work can flow in and out of the heat bath [12]. Further, if the manifold is assumed to be spin systems with particular temperature gradients, thermodynamic machines can be facilitated [13, 14].

In the following sections, we show that, in a cavity optomechanical system, these thermodynamic processes can be realized on the cavity field. When the mirror is modeled as a multi-mode quantum oscillator, it can play the role of heat bath that thermalizes the cavity system according to the spectral density of states of the modeled oscillator. In other words, when the spectral density is so specified, the ensemble average energy of the cavity system evolves over time in the form as a square wave, giving off an on-off effect about the interaction between the system and the heat bath. The jumping of the energy up and down on the square wave matches with the diabatic processes during which heat is either infused into or extracted from the system. The time during which the energy stays fixed designates the adiabatic processes where no heat is transferred but work is done on the cavity field.

To understand this complex process more clearly, we start our discussion of quantum thermodynamics in Sec. 2 below by studying the simple interaction between a single-mode oscillator and a two-level system. The ensemble average energy of the two-level system is shown to be oscillating. The study is then expanded to the interaction between a multi-mode oscillator and a two-level system in Sec. 3, in which the on-off interaction effect is demonstrated. Thereafter, modeling the heat bath on the multi-mode oscillator, the four processes of the thermodynamic cycle are identified and the relevant spectral density is given in Sec. 4. The conclusion is given in Sec. 5.

2. Single-mode oscillating effect

We consider a two-level system $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$ as the main system and a single-mode oscillator $\{a, a^\dagger\}$ as the controller with a heat reservoir. Let their interaction be

the usual dipole-field coupling, then the total Hamiltonian is ($\hbar = 1$)

$$H = \Omega\sigma_z + \omega a^\dagger a + \eta(a + a^\dagger)\sigma_z. \quad (1)$$

Further, let $|\psi_n^e\rangle$ ($|\psi_n^g\rangle$) be the eigenstate of the controller associated with the excited state $|e\rangle$ (ground state $|g\rangle$) of the system, where n designates the Fock number of the oscillator. The tensor product $|e, \psi_n^e\rangle = |e\rangle \otimes |\psi_n^e\rangle$ describe the eigenstate of the combined system and controller. Applying the Hamiltonian (1) to this product state, we get

$$\begin{aligned} H |e, \psi_n^e\rangle &= [\Omega + \omega a^\dagger a + \eta(a + a^\dagger)] |e, \psi_n^e\rangle \\ &= \left[\Omega + \omega \left(a^\dagger + \frac{\eta}{\omega} \right) \left(a + \frac{\eta}{\omega} \right) - \frac{\eta^2}{\omega} \right] |e, \psi_n^e\rangle. \end{aligned} \quad (2)$$

That means, when the system stays in the excited state, the part of Hamiltonian that determines the evolution of the controller is effectively a displaced oscillator

$$H^e = \omega A_e^\dagger A_e - \frac{\eta^2}{\omega} \quad (3)$$

where $A_e = a + \eta/\omega$. In other words, while interacting with the excited system, the eigenstate of the controller is a displaced Fock state: $|\psi_n^e\rangle = D(\frac{\eta}{\omega})|n\rangle$, where $D(\frac{\eta}{\omega}) = \exp\{\frac{\eta}{\omega}(a^\dagger - a)\}$ denotes the displacement operator. Associated with this eigenstate, the eigenvalue for the effective Hamiltonian (3) is then

$$H^e |\psi_n^e\rangle = \epsilon_n^e |\psi_n^e\rangle = \left(n\omega - \frac{\eta^2}{\omega} \right) |\psi_n^e\rangle \quad (4)$$

and the total eigenenergy of the combined system and controller is

$$E_n^e = \Omega + n\omega - \frac{\eta^2}{\omega}. \quad (5)$$

Following the same considerations, the system ground state $|g\rangle$ is associated with the inversely displaced Fock state $|\psi_n^g\rangle = D(-\frac{\eta}{\omega})|n\rangle$ of the controller. The effective Hamiltonian for the controller is

$$H^g = \omega A_g^\dagger A_g - \frac{\eta^2}{\omega} \quad (6)$$

with $A_g = a - \eta/\omega$, for which the total eigenenergy differs from the excited state only by the sign of the system eigenenergy, i.e.

$$E_n^g = -\Omega + n\omega - \frac{\eta^2}{\omega}. \quad (7)$$

We can now consider the system dynamics for its coupling to the controller thermal reservoir. Assume that the initial state has the controller retain a Fock number n and the corresponding density matrix is in a thermal equilibrium with Bernoulli distribution

$$\rho(0) = P_e |e, \psi_n^e(0)\rangle \langle e, \psi_n^e(0)| + P_g |g, \psi_n^g(0)\rangle \langle g, \psi_n^g(0)|. \quad (8)$$

The Hamiltonian (1) drives the evolution of the system and the controller separately according to what we discussed above, i.e.

$$\begin{aligned} \rho(t) &= e^{-iHt} \rho(0) e^{iHt} \\ &= P_e |e, \psi_n^e(t)\rangle \langle e, \psi_n^e(t)| + P_g |g, \psi_n^g(t)\rangle \langle g, \psi_n^g(t)| \end{aligned} \quad (9)$$

where we have denoted $|\psi_n^e(t)\rangle = e^{-iH^e t} |\psi_n^e(0)\rangle$ and $|\psi_n^g(t)\rangle = e^{-iH^g t} |\psi_n^g(0)\rangle$.

The energies stored in the system S and the controller C varies with time according to the initial Bernoulli distribution and the system parameters but their total energy remains static if the density matrix starts off from the initial state given in Eq. (8). Taking $\{|e\rangle, |g\rangle\}$ as the basis of the system and $\{|\psi_n^e\rangle, |\psi_n^g\rangle\}$ as the basis of the controller, we can verify the constancy of the total energy, i.e.

$$\begin{aligned}\langle H(t) \rangle &= \text{tr}_{S+C} (\rho(t) [\Omega \sigma_z + \omega a^\dagger a + \eta(a + a^\dagger) \sigma_z]) \\ &= \Omega(P_e - P_g) + P_e \epsilon_n^e + P_g \epsilon_n^g.\end{aligned}\quad (10)$$

However, since the controller acting as a heat reservoir has constant influx or outflux of thermal energy to and from the two-level system, the energy of the system and its interaction with the controller will not remain constant. The average taken over the reduced density matrix of the controller gives

$$\begin{aligned}\langle \Omega \sigma_z + \eta(a + a^\dagger) \sigma_z \rangle_C &= P_e \langle \psi_n^e | \Omega + \eta(a_e(t) + a_e^\dagger(t)) | \psi_n^e \rangle \sigma_z \\ &\quad + P_g \langle \psi_n^g | \Omega + \eta(a_g(t) + a_g^\dagger(t)) | \psi_n^g \rangle \sigma_z,\end{aligned}\quad (11)$$

where $a_e(t)$ and $a_g(t)$ are the annihilation operators in the Heisenberg picture

$$a_e(t) = e^{iH^e t} a e^{-iH^e t}, \quad (12)$$

$$a_g(t) = e^{iH^g t} a e^{-iH^g t}. \quad (13)$$

of the controller following the evolutions of the excited state and ground state respectively. The Hamiltonians (3) and (6) can be then considered as what are effectively driving the dynamics of the controller since $[H(t), a_e(t)] = [H^e(t), a_e(t)]$ and $[H(t), a_g(t)] = [H^g(t), a_g(t)]$.

The time-dependent operators can be expressed explicitly by their Heisenberg equations with respect to these Hamiltonians:

$$\dot{a}_e(t) = -i\omega a_e(t) - i\eta, \quad (14)$$

$$\dot{a}_g(t) = -i\omega a_g(t) + i\eta, \quad (15)$$

for which the system-controller interaction η is essentially a driving of the level populations towards opposite directions for the two system levels. We will see in the next section that this driving translates to energy transfers in and out of the system levels. Substituting the formal solutions to Eqs. (14)-(15) into (11), we find

$$\begin{aligned}\langle \Omega \sigma_z + \eta(a + a^\dagger) \sigma_z \rangle_C &= \Omega \sigma_z + \\ &\quad \sum_{\gamma \in \{e, g\}} P_\gamma \eta \langle \psi_n^\gamma | a(0) e^{-i\omega t} + a^\dagger(0) e^{i\omega t} + (-1)^\gamma \frac{2\eta}{\omega} (1 - e^{-i\omega t}) | \psi_n^\gamma \rangle\end{aligned}\quad (16)$$

where we let γ be either 1 to indicate the positive sign for the excited state or 0 to indicate the negative sign for the ground state. The ensemble average of the operator a^\dagger at the initial state can be written as a c-number with amplitude α and phase ϕ

$$\langle a^\dagger(0) \rangle = \langle \psi_n^e | a^\dagger(0) | \psi_n^e \rangle + \langle \psi_n^g | a^\dagger(0) | \psi_n^g \rangle = \alpha \cos \phi. \quad (17)$$

Therefore, the average energy $\langle \Omega \sigma_z + \eta(a + a^\dagger) \sigma_z \rangle_C$ becomes an oscillating value

$$\left[\Omega + 2\eta\alpha \cos(\omega t - \phi) + \frac{2\eta^2}{\omega} (P_e - P_g)(\cos \omega t - 1) \right] \sigma_z, \quad (18)$$

where the direction of the oscillation depends on the system state.

3. Multi-mode square wave on-off effect

We now extend the concepts introduced in the last section to the case of a multi-mode oscillator. The Hamiltonian (1) becomes

$$H = \Omega\sigma_z + \sum_j \omega_j a_j^\dagger a_j + \sum_j \eta_j (a_j + a_j^\dagger)\sigma_z, \quad (19)$$

where the system part $H_S = \Omega\sigma_z$ stays identical while the controller part $H_C = \sum_j \omega_j a_j^\dagger a_j$ and the interaction part $H_I = \sum_j \eta_j (a_j + a_j^\dagger)\sigma_z$ extends to the summation over all modes indexed by j . The associated eigenstate for the excited system becomes a tensor product

$$\left| e, \left\{ \psi_{n_j}^e \right\} \right\rangle = |e\rangle \prod_{\otimes j} \left| \psi_{n_j}^e \right\rangle \quad (20)$$

over the Fock states of all the photonic modes of the field.

Applying Eq. (19) to the eigenstate, we find the effective Hamiltonian for the multi-mode controller $\left| \left\{ \psi_{n_j}^e \right\} \right\rangle$ to be

$$H^e = \sum_j \left(\omega_j A_{e,j}^\dagger A_{e,j} - \frac{\eta_j^2}{\omega_j} \right), \quad (21)$$

where the displaced annihilation operator is now distinguished for each mode

$$A_{e,j} = D^{-1} \left(\frac{\eta_j}{\omega_j} \right) a_j D \left(\frac{\eta_j}{\omega_j} \right). \quad (22)$$

An identical procedure can be applied to the ground state, for which the index e in the equations above will be replaced by g .

When the two-level system retains its Bernoulli distribution, the associated density matrix here only differs from Eq. (9) by the expressions in the eigenstates, i.e. we can verify

$$\begin{aligned} \rho(t) = & P_e \left| e, \left\{ \psi_{n_j}^e(t) \right\} \right\rangle \left\langle e, \left\{ \psi_{n_j}^e(t) \right\} \right| \\ & + P_g \left| g, \left\{ \psi_{n_j}^g(t) \right\} \right\rangle \left\langle g, \left\{ \psi_{n_j}^g(t) \right\} \right| \end{aligned} \quad (23)$$

where

$$\left| e, \left\{ \psi_{n_j}^e(t) \right\} \right\rangle = |e\rangle \otimes e^{-iH^e t} \left| \left\{ \psi_{n_j}^e \right\} \right\rangle \quad (24)$$

and similarly for the ground state.

Again, to find the average energy over time for arbitrary system distributions, we consider the evolution of the operators

$$a_{e,j}(t) = e^{iH^e t} a_j e^{-iH^e t}, \quad (25)$$

$$a_{g,j}(t) = e^{iH^g t} a_j e^{-iH^g t}. \quad (26)$$

and their adjoints under the Heisenberg picture. Since the multiple modes of the oscillator are orthogonal, it's obvious we can generalize Eqs. (14)-(15) to

$$\dot{a}_{e,j}(t) = -i\omega a_{e,j}(t) - i\eta_j, \quad (27)$$

$$\dot{a}_{g,j}(t) = -i\omega a_{g,j}(t) + i\eta_j. \quad (28)$$

The evolutions according to these equations will give rise to the average energy

$$\begin{aligned} \langle H_S + H_I \rangle_C = & \left[\Omega + \sum_j 2\eta_j \left(\alpha_j \cos(\omega_j t - \phi_j) \right. \right. \\ & \left. \left. + \frac{\eta_j}{\omega_j} (P_e - P_g) (\cos \omega_j t - 1) \right) \right] \sigma_z \end{aligned} \quad (29)$$

over the orthogonal controller basis $\{|\psi_{n_j}^e\rangle, |\psi_{n_j}^g\rangle\}$. Similarly, α_j and ϕ_j are the initial amplitude and phase of the j -th mode.

Eq. (29) looks like a simple extension to Eq. (18) in its form. However, by reshuffling the terms, we can arrange it to become a Fourier series about time t with nonzero coefficients in both the sines and the cosines

$$\begin{aligned} \langle H_S + H_I \rangle_C = \sigma_z \Bigg\{ & \Omega - (P_e - P_g) \sum_j \frac{2\eta_j^2}{\omega_j} + \sum_j 2\eta_j \\ & \left[\left(\alpha_j \cos \phi_j + \frac{\eta_j}{\omega_j} (P_e - P_g) \right) \cos \omega_j t \right. \\ & \left. + \alpha_j \sin \phi_j \sin \omega_j t \right] \Bigg\}. \end{aligned} \quad (30)$$

When setting this Fourier series with different coefficients, we can obtain different cyclic waves. In other words, the multi-mode coupling between the oscillator as the controller and the two-level system has given us an edge of control over the average energy stored in the system by setting different initial states for the controller.

The typical case is when setting the Fourier series as a square wave, for which the on-off switching of the stored energy occurs. Consider that we let

$$\begin{cases} 2\eta_j \alpha_j \sin \phi_j = \frac{A}{\omega_j} \\ 2\eta_j \left(\alpha_j \cos \phi_j + (P_e - P_g) \frac{\eta_j}{\omega_j} \right) = 0 \end{cases} \quad (31)$$

to furnish a sinc function for the sine series and zero out the cosine series. Then the square wave of height A would be realized if $\omega_j = (2j - 1)\omega_0$ are the odd harmonics of some fundamental frequency ω_0 and

$$\alpha_j = \frac{\sqrt{4(P_e - P_g)^2 \eta_j^4 + A^2}}{2\eta_j(2j - 1)\omega_0}, \quad (32)$$

$$\phi_j = -\tan^{-1} \left(\frac{A}{2(P_e - P_g)\eta_j^2} \right). \quad (33)$$

We can observe we conclude that the amplitudes α_j determines the frequency of the Fourier series while the phases ϕ_j determines the height of the square wave or the amount of energy being transferred in and out of the system.

Finally, following the nomenclature of quantum thermodynamics, we can define a spectral density function

$$J(\omega) = \sum_j \frac{4(P_e - P_g)^2 \eta_j^4 + A^2}{4\eta_j^2 \omega_j^2} \delta(\omega - \omega_j) \quad (34)$$

for the oscillator controller as a heat reservoir. This system-specific reservoir will supply energy to the system such that it will undergo energy cycling with the system

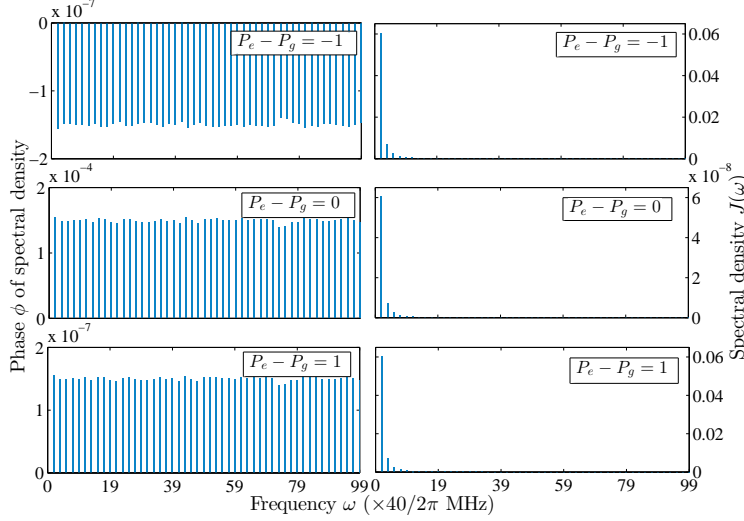


Figure 1. Spectral density distributions $J(\omega)$ for the first 50 odd harmonics of base frequency ω_0 with population inversion $P_e - P_g$ at no inversion, half inversion, and full inversion, respectively.

at period $T = 2\pi/\omega_0$, during which half of the time the system will be turned on to attain a higher energy and half of the time the system will be turned off to a lower energy state. The distribution of the spectral density depends on the coupling strengths η_j as well as the desired population distribution $\{P_e, P_g\}$ of the two-level systems.

In Fig. 1, the spectral density is plotted for three values of $P_e - P_g$: -1 for no population inversion, 0.001 for half inversion (uniform Bernoulli distribution), and 1 for full population inversion. The targeted amplitude A is $30/2\pi$ MHz. The base frequency ω_0 is set the experimentally accessible detuning of $40/2\pi$ MHz between the cavity and a driving laser. The first fifty odd harmonics over this base frequency are considered for the heat reservoir. We assume the spectral density of the coupling strength adopt a normal distribution with variance $100/2\pi$ kHz about the mean $10/2\pi$ MHz. We can notice that to realize the on-off switching effect, the oscillator is generally not very difficult to prepare. No matter what the amount of inversion the two-level system retains, the associated spectral density of the oscillator as a heat reservoir has fairly uniform phase distributions. As for the magnitude $J(\omega)$, we observe it is sufficient to produce only the first few harmonics for the purpose of heat transfer. In particular, for the half-inversion case, the amplitude of $J(\omega)$ is almost negligible across all harmonics since the uniform Bernoulli distribution at the two-level system naturally induces the energy exchange with the oscillator controller.

4. Thermodynamic Cycle of cavity optomechanical system

With the on-off effect shown on an oscillator-coupling two-level system, we now turn eventually to the study of a movable mirror in an optomechanical cavity associated with the thermodynamic energy transfer. The previously studied two-level system σ_z is replaced here by a pair of annihilation and creation operators $\{b, b^\dagger\}$ to represent

an optical field traversed in the cavity. The effective dynamics of the movable mirror mounted on a cantilever is determined by the flexibility modulus of the cantilever materials. It is modeled by a multi-mode oscillator with Hamiltonian $\sum_j \omega_j a_j^\dagger a_j$ and can be regarded as a heat bath under the frameworks of Feynman-Vernon [15] and Caldeira-Leggett [16].

The motion of the mirror deforms the cavity volume, which results in a radiation pressure proportional to the cavity photon number $b^\dagger b$ and the mirror displacement x being feedback to the mirror [1]. Expressing the displacement x in terms of the canonical conjugate variables a_j and a_j^\dagger , the total Hamiltonian reads

$$\begin{aligned} H &= H_S + H_C + H_I \\ &= \Omega b^\dagger b + \sum_j \omega_j a_j^\dagger a_j + b^\dagger b \sum_j \eta_j (a_j + a_j^\dagger). \end{aligned} \quad (35)$$

To examine the heat exchange process over time, we consider the density matrix

$$\rho(0) = \rho_S(0) \otimes \rho_C(0) = \sum_m P_m \left| m, \left\{ \psi_{n_j}^m \right\} \right\rangle \left\langle m, \left\{ \psi_{n_j}^m \right\} \right|. \quad (36)$$

to represent an initial mixed state at thermal equilibrium, where $|m\rangle$ is the Fock eigenstate for the photon number in the cavity and $|\psi_{n_j}^m\rangle$ is the associated eigenstate of the mirror controller. At time t , the effective Hamiltonian that drives the evolution of $|\psi_{n_j}^m\rangle$ is

$$H^m = m\Omega + \sum_j \omega_j a_j^\dagger a_j + m \sum_j \eta_j (a_j + a_j^\dagger), \quad (37)$$

which gives rise to the reduced density matrix of the mirror controller as

$$\rho_C(t) = \sum_m P_m e^{-iH^m t} \left| \left\{ \psi_{n_j}^m \right\} \right\rangle \left\langle \left\{ \psi_{n_j}^m \right\} \right| e^{iH^m t}. \quad (38)$$

Letting $\langle H_S(t) + H_I(t) \rangle_C = \Omega(t) b^\dagger b$, we find the effective eigenenergy of the cavity system to be

$$\Omega(t) = \Omega + \sum_m P_m \sum_j \eta_j \left\langle \left\{ \psi_{n_j}^m \right\} \right| \left[a_{m,j}(t) + a_{m,j}^\dagger(t) \right] \left| \left\{ \psi_{n_j}^m \right\} \right\rangle, \quad (39)$$

where $a_{m,j}(t)$ and $a_{m,j}^\dagger(t)$ are the evolved operators in the Heisenberg picture similar to those defined in Eqs. (25)-(26), except that the system states are here extended to all m levels.

Following the same routine of the last section but assuming a continuous spectrum for the mirror as a heat reservoir, we would arrive at

$$\Omega(t) = \Omega_0 - 2 \int d\omega \frac{\eta}{\omega_0} \left\{ \left(\alpha \cos \phi + \frac{\mu \eta}{\omega} \right) \cos \omega t + \alpha \sin \phi \sin \omega t \right\}, \quad (40)$$

where $\Omega_0 = \Omega - 2\mu \int (\eta^2/\omega) d\omega$ is the renormalized energy offset when the cavity interacts with the heat reservoir. In the equation, $\mu = \sum_m P_m m$ is the weighted mean of the photon population across all levels in the cavity system. To furnish the on-off effect studied in the last section, the phase and magnitude of the spectral density should, therefore, be

$$\phi(\omega) = -\tan^{-1} \frac{A\omega_0}{2\mu\eta^2}, \quad (41)$$

$$J(\omega) = \frac{4\mu^2\eta^4 + A^2\omega_0^2}{4\eta^2\omega^2}. \quad (42)$$

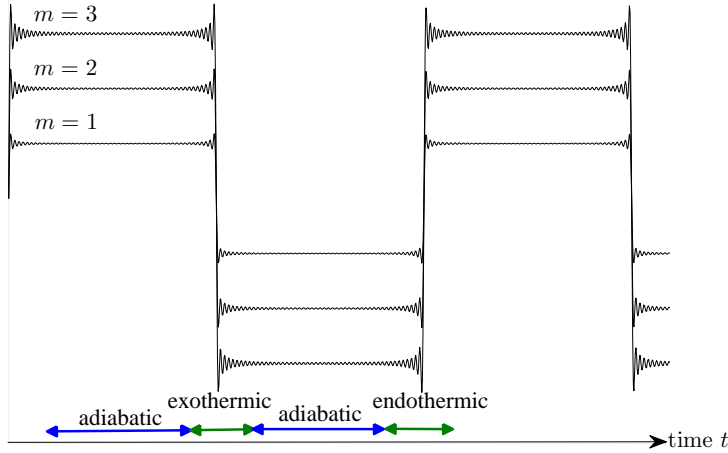


Figure 2. (Color online) Level diagrams of the first three Fock states of the cavity system over time, exhibiting a cyclic energy transfer between the system and the reservoir. Within each cycle, four thermodynamic stroke processes can be identified.

If the spectral density is set so, $\Omega(t)$ will cycle about Ω_0 with amplitude A between two constant values, as illustrated in Fig. 2, where the reservoir is again assumed to consist of 50 odd harmonics over a base frequency. The durations within which $\Omega(t)$ stays constant can be identified with adiabatic processes. During these processes, though the cavity remains interacting with the reservoir, no energy is transferred into or out of the cavity and all levels m remain constantly spaced. Between these processes, the cavity system either absorbs energy from the reservoir, raising up all levels simultaneously, or infuses energy back to the reservoir, letting itself fall back to the original levels. The absorption of energy can be identified, thermodynamically, with an endothermic process whereas the depletion of energy can be identified with an exothermic process.

5. Conclusion

We have shown a specific thermodynamic cycle on a cavity optomechanical system. By first demonstrating the cyclic eigenenergy of a two-level system interacting with a single-mode oscillator, we then prove that within the cavity system, the cavity field can act as the thermal system relative to the multi-mode oscillating mirror acting as the heat reservoir that controls the energy flow. By matching the spectral density of the mirror with that of a square wave in the cyclic eigenenergy, an on-off cycle of energy exchange can be constructed, during which four thermodynamic stroke processes can be identified.

Therefore, a quantum mechanical system with appropriate spectral densities can serve as a quantum thermodynamic machine. Future investigations will focus on how the constructed thermalization processes fit within the general framework of quantum heat engines.

Acknowledgments

The author thanks the support by FDCT of Macau under grant 013/2013/A1 and by University of Macau under grant MRG022/IH/2013/FST.

References

- [1] P. Meystre, E. M. Wright, J. D. McCullen, and E. Vignes, J. Opt. Soc. Am. B 2, 1830 (1985).
- [2] T. J. Kippenberg and K. J. Vahala, Science 321, 1172 (2008).
- [3] C. H. Metzger and K. Karrai, Nature 432, 1002 (2004).
- [4] S. Gigan, H. R. Bhm, M. Paternostro, F. Blaser, G. Langer, J. B. Hertzberg, K. C. Schwab, D. Buerle, M. Aspelmeyer, and A. Zeilinger, Nature 444, 67 (2006).
- [5] O. Arcizet, P.-F. Cohadon, T. Briant, M. Pinard, and A. Heidmann, Nature 444, 71 (2006).
- [6] H. Ian, Z. Gong, and C. Sun, Front. Phys. China 3, 294 (2008).
- [7] M. H. Schleier-Smith, I. D. Leroux, H. Zhang, M. A. Van Camp, and V. Vuletić, Phys. Rev. Lett. 107, 143005 (2011).
- [8] H. Ian, Z. R. Gong, Y. Liu, C. P. Sun, and F. Nori, Phys. Rev. A 78, 013824 (2008).
- [9] Z. R. Gong, H. Ian, Y. Liu, C. P. Sun, and F. Nori, Phys. Rev. A 80, 065801 (2009).
- [10] M. Wallquist, K. Hammerer, P. Zoller, C. Genes, M. Ludwig, F. Marquardt, P. Treutlein, J. Ye, and H. J. Kimble, Phys. Rev. A 81, 023816 (2010).
- [11] C. Joshi, J. Larson, M. Jonson, E. Andersson, and P. hberg, Phys. Rev. A 85, 033805 (2012).
- [12] H. T. Quan, P. Zhang, and C. P. Sun, Phys. Rev. E 72, 056110 (2005).
- [13] F. Tonner and G. Mahler, Phys. Rev. E 72, 066118 (2005).
- [14] M. Youssef, G. Mahler, and A.-S. F. Obada, Phys. Rev. E 80, 061129 (2009).
- [15] R. Feynman and F. Vernon Jr., Annals of Physics 24, 118 (1963).
- [16] A. O. Caldeira and A. J. Leggett, Phys. Rev. Lett. 46, 211 (1981).